Math Required

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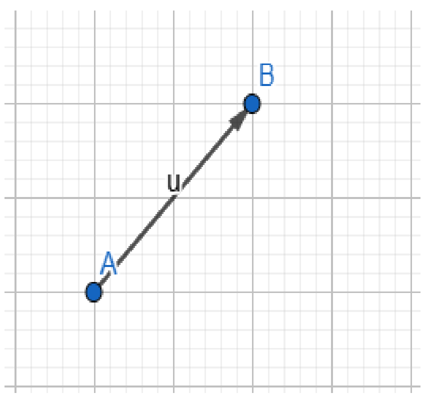
Outline

- [ ] [Vectors]Be able to understand the principle of Vector addition
- [ ] [Vectors]Be able to understand that a vector in 2 Dimensions can comprise of the sum of two mutually perpendicular vectors
- [ ] [Vectors]Be able to resolve vectors given an angle
- [ ] Small Angle approximation

[O Level E.Math]

A vector has both **magnitude and direction**.



The magnitude is represented by length AB, while the direction is represented by the arrow pointing from A to B.

A vector, **AB**, represents getting from Point A to Point B. (regardless of path taken), in the direction AB.

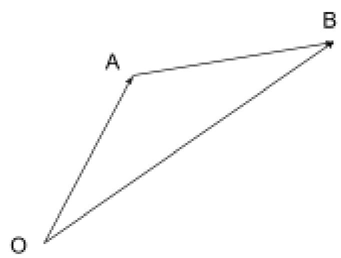
Figure 1. [1]

Vector Addition [EMATH Recap!]

An infinite number of vectors can be added together, to give a resultant. For simplicity sake (and o level emath) we'll be dealing with 2 vectors.

$$\overline{AB} = \overline{AO} + \overline{OB}$$

Note: for simplicity sake, we would **bold** the vector, in this document. Vectors can also be represented with an arrow above it or a squiggly line below the vector.

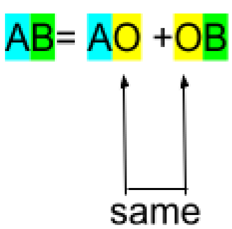


Diagrammatically:
Since **OA = -AO**

**To go in the reverse direction, just add a negative sign.

AB means from A to B.
To get to B from A, we move towards O, then towards B, and this would be the mathematically equivalent to AB.
Ie. the sum of vectors **AO** and **OB** is equal to that of **AB**.

Hence,



In O levels you'll learn this formula for E.Math, and for physics, you'd have learnt vector addition of forces using the tip to tail, or parallelogram method.

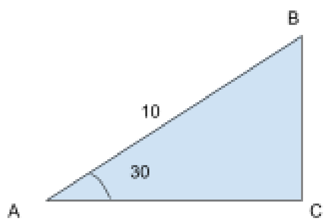
But what if you have **more than 2 forces?** What then, do you apply the tip to tail method twice? Or do you find some reasoning to cancel out forces? The answer is no.

For multiple forces acting on an object, we would **resolve forces** into the **x and y direction**.

Vector Resolving

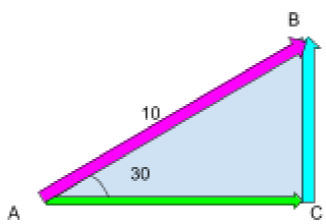
Example 1a: [Simple O level trigonometry].

Let's say you have a right angled triangle, with angle BAC=30 degrees.
If the hypotenuse of the triangle is 10 units, what is length AC and BC respectively?



Answer:

Similarly if AB was now a vector **AB** of magnitude 10 units, length AC would represent the magnitude of the component of the vector acting in the X direction, while length BC would represent the magnitude of the component of the vector in the Y direction.



This shows how a vector can be mathematically represented as two mutually perpendicular vectors in the horizontal and vertical directions

Note: it may not always be X or Y.

Example 1b: Force Acting at an Angle

If a 20N force was acting on a block at 45 degrees to the horizontal, what is the magnitude of the force acting in the horizontal and vertical directions respectively?

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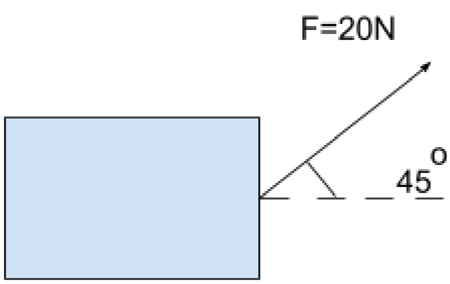
Answer:



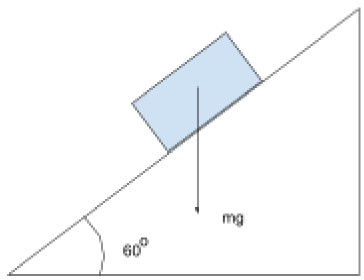
Vector Resolving in Non x-y direction

Example 2a: Resolving of Forces at an incline.

Given that the slope is inclined at 60 degrees, what is the component of the box's weight acting:

(a) In the direction parallel to the slope?
(b) In the direction perpendicular to the incline of the slope?

[Express all answers in terms of mg]



Small Angle Approximation

Mostly useful for understanding of simple harmonic motion equations (SHM)

$$\sin x \approx x$$
$$\cos x \approx 1 - \frac{x^2}{2}$$
$$\tan x \approx x$$

The most important is sin(x),.

Extra Practice Questions [More can be found under Forces]

1. A force F1 of magnitude 6.00 units acts on an object at the origin in a direction θ=30.0° above the positive x axis). A second force F2 of magnitude 5.00 units acts on the object in the direction of the positive y axis. Find the magnitude and direction of the resultant force F1 and F2. [Serway, 3.2.10] Try using vector resolving, even if it may be possible to solve using accurate drawing!
2. ***[More challenging] A snow-covered ski slope makes an angle of 35.0° with the horizontal. When a ski jumper plummets onto the hill, a parcel of splashed snow is thrown up to a maximum displacement of 1.50 m at 16.0° from the vertical in the uphill direction as shown in Figure P3.28. Find the components of its maximum displacement. [Serway, 3.4.28]

- (a) parallel to the surface and
- (b) perpendicular to the surface

References:

[1] Figure 1. GeoGebra.Interactive Mathematics Visualization. Retrieved from https://www.geogebra.org/m/bAnquQpH.

Appendix A: Implicit Differentiation

Basics of Implicit Differentiation

A very powerful tool when solving physics questions, and it is used for many formula derivations. It'll be good to know and appreciate.

Given $y^2 = x + 2$, and asked to solve for $\frac{dy}{dx}$, instead of expressing x in terms of y, implicit differentiation can be used to solve for $\frac{dy}{dx}$ directly.

Step 1: Differentiate the whole expression with respect to x.

$2y\frac{dy}{dc} = 1$

Treat 'y' like you would treat x, and apply the rules of differentiation. But after differentiating y, add a $\frac{dy}{dx}$ term afterwards. Differentiating LHS, we use the power rule, bringing down the two, so the coefficient of y becomes 1. Since we differentiated y, we add $\frac{dy}{dx}$ next to y.

Displacement, velocity, acceleration.

Displacement = s
Since $v = \frac{ds}{dt}$ ie. velocity is the rate of change in displacement wrt time, t.

As we implicitly differentiate s,
we bring down the power 1, and minus 1 from the coefficient → $1(s)^{1-1}$ → $1(s)^0$
Then we add $\frac{ds}{dt}$ after 's' → $1(s)^0\frac{ds}{dt}$→ $\frac{ds}{dt}$

Hence, we get $v = \frac{ds}{dt}$. A similar approach is used to get from velocity → acceleration.

$v = \frac{ds}{dt}$

$a = \frac{d^2s}{dt^2} = \frac{dv}{dt}$

[Note: not very relevant to solving problems but useful for understanding equations]

A very cool proof of implicit differentiation :
https://www.mathsisfun.com/calculus/implicit-differentiation.html